Power of Averaging

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Averaging *has many simple and familiar forms widely being used in trading stocks or other investments to better combat market uncertainties. Instead of buying or selling the whole position at once, averaging techniques use multiple orders to establish or liquidate a position to reduce volatility. Unfortunately, the answers to some fundamental questions on averaging are not well understood today. Averaging up is better or averaging down is better? How can averaging help in taking profit or cutting loss? Is an up or down market making any difference in selecting an averaging strategy?*

Introduction

Averaging is a popular strategy for both traders and investors. One most accepted form of averaging, known as dollar averaging or dollar cost averaging, was first researched long ago by Dr. Leonard W. Ascher in 1960 [1]. Since then, more studies have been done on dollar cost averaging [2] – [9] with emphasis on averaging as a long term investment strategy. However, most research in the past did not address important and practical cases involving selling and multiple positions. Even on a more basic level, misconceptions and misuse of averaging techniques still exist among traders, investors, and even financial institutions. For example, financial institutions still state that "dollar cost averaging may reduce your average cost of investing over time" [10]. In fact, because securities' prices never remain constant over time, it can be mathematically proved that "dollar cost averaging always reduces your average cost of investing over time". It is a big difference between "may" and "always" in its impact on investor or trader confidence.

The purpose of this paper is to present a foundation for analyzing and evaluating averaging techniques for improving trading profit or return on investment. Three of the most popular averaging techniques are formulated, analyzed, and compared. We will define the problem first for studying averaging techniques so that all averaging techniques can be compared on the same basis. A basic mathematical principle behind averaging is summarized to clarify the misconception on dollar cost averaging. We will then evaluate trading power of averaging for long or short positions as well as up and down markets. As a more challenging research topic, we will extend this research to study the power of averaging for a typical multiple positions scenario. A mathematical proof is derived to show that through rebalancing when buying, additional averaging power can be obtained to yield better performance than dollar averaging.

Dollar Averaging (DA)

Among different averaging techniques, long term investors often use an investment strategy called dollar cost averaging designed to reduce volatility in which *securities, typically mutual funds, are purchased in fixed dollar amounts at regular intervals*, regardless of what direction the market is moving. For simplicity, we will use the term of Dollar Averaging (DA) instead of Dollar Cost Averaging in this paper. An example of DA is illustrated in Table 1.

Table 1

Dollar Averaging reduces average cost of investing

Time Period	Fixed Dollar	Share Price	Shares Purchased
1	$400	$10	40
2	$400	$8	50
3	$400	$5	80
4	$400	$8	50
5	$400	$10	40
Total	$2,000	$41	260

Average Share Price = ($10+$8+$5+$8+$10) / 5 = $8.20

Average Share Cost Paid = $2,000 / 260 = $7.69

Share Averaging (SA)

A more popular form of averaging is often used by traders, it is defined as Share Averaging (SA) in this paper, meaning *a fixed amount of shares (not dollar) are bought or sold at selected time points*. Traders use SA in a so-called "averaging down" technique for share accumulation - buying additional shares of a stock after a position is already established, also which has dropped in price since the earlier purchase. Another practical form of SA is used by corporate insiders who have set up automatic programs to sell constant shares of their company stock each quarter as a way of diversifying their holdings. An example of SA is illustrated in Table 2.

Table 2

Share Averaging does not reduce average cost of investing

Time Period	Share Price	Fixed Share	Dollar Invested
1	$10	50	$500
2	$8	50	$400
3	$5	50	$250
4	$8	50	$400
5	$10	50	$500
Total	$41	250	$2,050

Average Share Price = $41 / 5 = $8.20

Average Share Cost Paid = $2,050 / 250 = $8.20

Ratio Averaging (RA)

The third form of averaging studied in this paper is Ratio Averaging (RA). RA is defined as *an enhanced form of DA with rebalancing over time to maintain a constant ratio for multiple positions.* For example, when RA is applied to two positions, say stock ABC and stock XYZ, an initial ratio of 50/50 is established and only half of total fund is used for establishing the initial positions. ABC and XYZ prices change over time. The remaining half of total fund is added for ratio averaging, i.e., 50/50 positions are reestablished. RA is applied to rebalancing the positions of ABC and XYZ to keep the 50/50 ratio constant.

A simple example of RA is illustrated in Table 4. The example for two positions using DA is given first in Table 3 as a benchmark for comparison.

Table 3

Dollar Averaging
– Two Positions ($400 each position per time period)

Time Period	Fixed Dollar	Share Price ABC	Share Price XYZ	ABC Shares Purchased	XYZ Shares Purchased
1	$800	$10	$10	40	40
2	$800	$8	$5	50	80
3	$800	$10	$10	40	40
Total	$2,400			130	160

ABC Average Share Price = $28 / 3 = $9.33
XYZ Average Share Price = $25 / 5 = $8.33
ABC Average Share Cost Paid = $1,200 / 130 = $9.23
XYZ Average Share Cost Paid = $1,200 / 160 = $7.5

Table 4

Ratio Averaging
– Two Positions ($800 invested per time period, 50/50 ratio each time)

Time Period	Share Price ABC	Share Price XYZ	Balance	ABC Shares Accumulated	XYZ Shares Accumulated
1	$10	$10	$800	40	40
2	$8	$5	$1,320	82.5	132
3	$10	$10	$2,940	147	147
Total			$2,940	147	147

ABC Average Share Price = $28 / 3 = $9.33
XYZ Average Share Price = $25 / 5 = $8.33
ABC Average Share Cost Paid = (40x$10 + 42.5x$8 + 64.5x$10) / 147 = $9.42
XYZ Average Share Cost Paid = (40x$10 + 72x$5 + 15x$10) / 147 = $6.19

As seen from the above examples, RA yields higher cost per share for ABC than DA but lower cost per share for XYZ. However, since the total accumulated shares using RA is more than DA, the net cost per share for RA is $2,400 / 294 = $8.16 versus the net cost per share for DA is $2,400 / 290 = $8.28. RA has yielded better performance than DA.

Under certain typical assumptions, we are going to prove that for two positions with averaging (buying), RA will always yield better performance than DA.

Target Price and Problem Formation

The continued search and recent development in various strategies based on technical analysis and fundamental analysis for buying and selling stocks has intensified the interest in forming a foundation to compare different strategies on a common basis.

The challenge of investing (long term) or trading (short term) is to find a lucrative buying and selling strategy. Given a starting position price and a projected target price, optimal time and the right execution price are essential in an averaging strategy to achieve maximum profit when target price is reached with minimum probability of incurring a loss.

Four key elements of an averaging strategy are identified as follows:

1. *Define a target price* - If the target price is higher than the start price, a buy strategy is formed. Otherwise, a short or sell strategy is considered necessary to short sell or liquidate all positions for the purpose of taking profits or cutting losses.
2. *Design a trading strategy* - A strategy can be any averaging technique such as DA, SA, RA, or even buy-hold-sell as its simplest form. A suitable strategy candidate must be practical for implementation.
3. *Evaluate the selected strategy* - Based on performance comparison, the best strategy is selected and then implemented.
4. *Adjust the selected strategy if necessary* - Should the target price is updated over time due to changing market conditions, all strategies may need to be reevaluated again to make sure the strategy implemented is still the best for the new target price.

The fundamental problem of investment, either short term trading or long term investing, is that of producing maximum profit through buying or selling a security (or securities) when a target price (or target prices) over a given time period is projected.

The above problem formulation is consistent with the widely adopted market analysts' research rating method which offers *12-Month Projected Target Prices* for the covered securities.

Often the amount of funds available for investment may not be large enough to neglect trade commissions or allow multiple positions. These size or trade commission aspects of investment are irrelevant, at least not the significant concerns, to the fundamental problem of investment. The significant aspect is that the best averaging technique is always selected and implemented to achieve maximum profit, and to reduce the risk of larger loss as possibly incurred in buying or selling the whole positions at once.

Harmonic Mean and Dollar Averaging

There are numerous methods for calculating the average of a set of *n* prices. For a given stock, if we buy or sell *n* times, the average prices, generally referred to simply as *the average*, is the arithnetic mean.

The harmonic mean is one of several methods of calculating an average. It is already known that the average cost per share given by Dollar Averaging is the harmonic mean of all prices [1]. Mathematically, *the harmonic mean is never larger than the arithmetic mean.*

To find the harmonic mean of a set of *n* prices, we add the reciprocals of all *n* prices, divide the sum by *n*, then take the reciprocal of the result.

For Dollar Averaging, we invest a fixed amount of money at regular intervals regardless of what direction the market is moving. By investing a fixed amount, we buy more shares when the price is low and fewer shares when the price is high. As a result, our average share cost will always be lower than the average share price that we actually paid. The "always' guarantee is given by the facts that (1) the harmonic mean is never larger than the arithmetic mean; and (2) share prices will never keep the same over time. This clarifies the misconception on dollar cost averaging among financial institutions and investors.

As an interesting remark, there is another popular mean called the geometric mean which is defined as the product of all the members of the number set, raised to a power equal to the reciprocal of the number of members. The geometric mean is useful to determine "average factors". For example, if a stock rose 10% in the first year, 20% in the second year and fell 15% in the third year, then we compute the geometric mean of the factors 1.10, 1.20 and 0.85 as $(1.10 \times 1.20 \times 0.85)^{1/3} = 1.0391...$ and we conclude that the stock rose on average 3.91 percent per year. The average of three year returns is (10% + 20% – 15%) / 3 = 5%. Similar to harmonic mean, the geometric mean of a data set is also always smaller than or equal to the set's arithmetic mean.

The arithmetic mean, the harmonic mean, and the geometric mean are equal if and only if all members of the data set are equal, which is never the case for any securities' market prices. In mathematics, using mathematical induction, it is not difficult to prove that harmonic mean is never larger than geometric mean, and geometric mean is never larger than arithmetic mean.

Power of Averaging

With averaging, multiple orders are applied to establishing or liquidating a position. This is in contrast to buying or selling the whole position at once as in a buy-hold-sell strategy. Depending on the market trend *up* or *down*, the cost per share using a buy-hold-sell strategy can be actually *lower* or *higher* than the average cost per share using an averaging strategy such as DA or SA. However, because of market price uncertainty, averaging techniques are often preferred to help reduce the scale of maximum possible loss if the market prices move in the opposite direction. This feature of better risk management is a basic attribute of averaging. The actual return on investment or trading profit further differentiates the power of averaging for different averaging strategies.

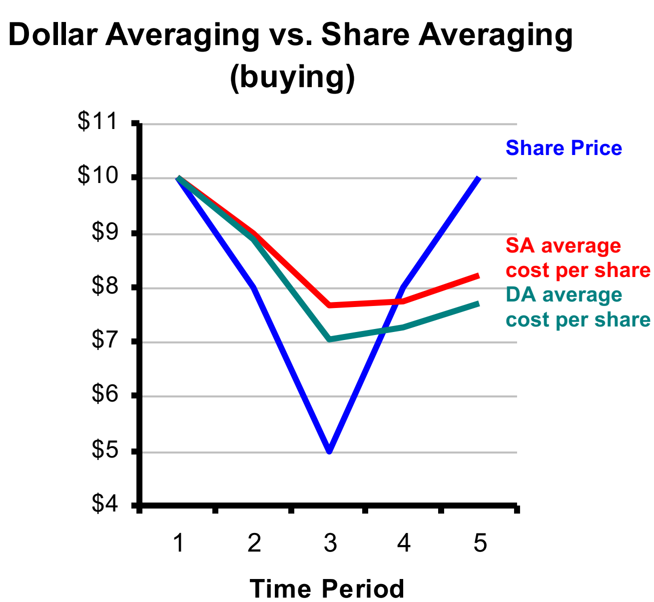
How well is one averaging technique compared to the other? In the following, we are comparing Dollar Averaging versus Share Averaging for buying, and for selling in particular, which has seldom studied in the past.

Averaging Techniques for Buying in an Up or Down Market

Market prices never remain constant. The harmonic mean as given by DA provides the guarrenteed lower average cost per share than the average price as given by SA. This conclusion is independent of up market or down market. In other words, if you are long, DA always beats SA. We thus conclude that DA has more buying power to make profit than SA.

Let us still use the example as given by Table 1 and make a share cost comparison for DA versus SA as shown in Figure 1. We can see that DA has the average cost per share always lower than the average cost per share of SA.

Figure 1



Averaging Techniques for Selling in an Up or Down Market

Now let us consider the power of averaging when selling. Because the harmonic mean is never larger than the arithmetic mean, *the average selling price given by Share Averaging is always higher than the average selling price if we use Dollar Averaging*. In other words, if you are selling or short, SA always beats DA. That means SA has more averaging power when selling to take profit or cut loss than DA. Again, this conclusion is independent of up market or down market.

Let us see an example of selling as given by Table 5. The result is shown in Figure 2. We assume that the intital investment starts with $2,000 value, i.e., 200 shares at $10 per share. The example shows that the average sold price using SA is $8.20 per share which is higher than $7.20 per share using DA. Thus SA is better than DA when selling.

Table 5

Share Averaging increases average sold price than Dollar Averaging

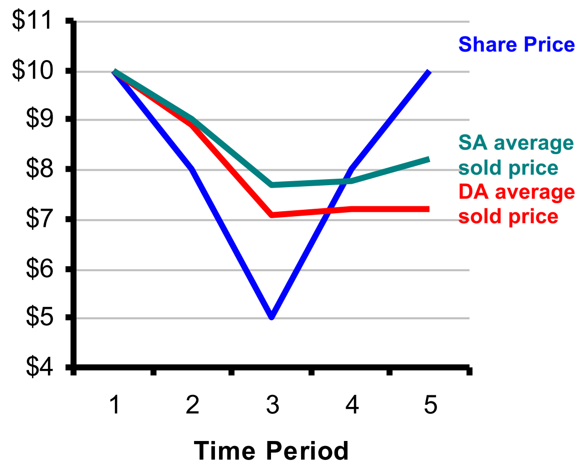
Time Period	Share Price	DA Shares Sold ($400 sold each time)	DA Cash Balance	SA Cash Balance (40 shares sold each time)
1	$10	40	$400	$400
2	$8	50	$800	$720
3	$5	80	$1,200	$920
4	$8	Only 30 shares left	$1,440	$1,240
5	$10	0	$1,440	$1,640
Total	$41	200	$1,440	$1,640

DA Average Sold Price = $1,440 / 200 = $7.20

SA Average Sold Price = $1,640 / 200 = $8.20

Figure 2

Share Averaging vs. Dollar Averaging (selling)



Cautions must be taken when we define a proper fixed dollar amount for DA in order to compare the result with SA on a common basis. For the example given in Table 5, we define $400 as the fixed amount for DA and 40 share each time for SA. It is shown that DA has sold all shares before SA has. The comparison results here can only be viewed as an approximate comparison.

In a down market, if DA uses a higher fixed dollar for selling such that DA has sold all shares before SA has, then DA could cut loss earlier to yield better performance than SA. However, this kind of exception should not be confused with the general conclusion that SA beats DA for selling. If a fixed dollar for DA and a fixed number of shares for SA remain unchanged for the whole selling period, SA will always have a higher average sold price than DA. This result is guaranteed by the mathematical principle about harmonic mean as described earlier.

SA is often used by many corporate insiders who have set up automatic programs to sell a fixed number of shares of their company stock each quarter as a way of diversifying their holdings. The above conclusion confirmed that SA is a better averaging strategy than DA for scheduled selling programs.

Averaging for Multiple Positions

Averaging techniques can also be applied to a portfolio of multiple positions to reduce losses should market prices move against your expected direction. Let us look at an example first.

The scenario under consideration has two postions, stock ABC and stock XYZ, with price changes as given in Table 6.

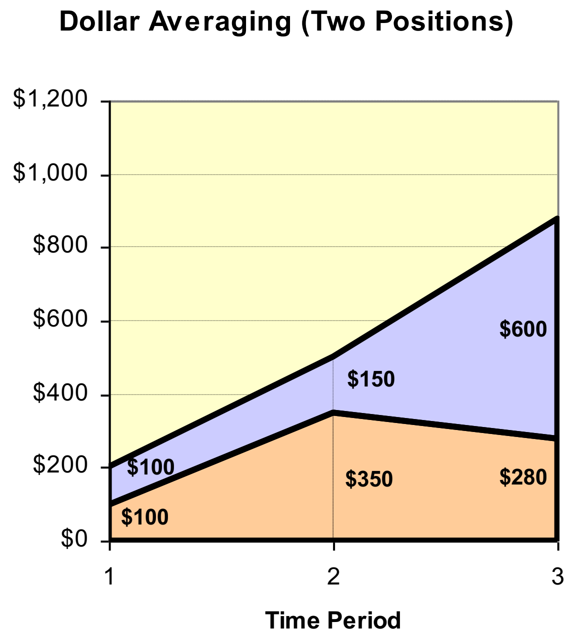
Table 6

Time Period	ABC Price	XYZ Price
1	$10	$10
2	$25	$5
3	$20	$20

In this example, we are assumed to have $400 initially. Averaging techniques are considered to help reduce volatility by investing $200 at time peorid 1 and then add the other $200 at time period 2. Both stocks have the starting prices of $10 and the target prices of $20.

Dollar Averaging (DA) - We use half of our fund ($200) to buy 10 shares of ABC and 10 shares of XYZ. At time period 2, ABC share rises to $25 and XYZ share declines to $5. With DA, we then use the other $200 to buy 4 shares of ABC and 20 shares of XYZ in a fixed dollar amount of $100 each position. When both stocks reach the target prices of $20, we end up with 14 shares of ABC and 30 shares of XYZ. Thus, the resulting profit is $480 and the value of each position is shown in Figure 3.

Figure 3



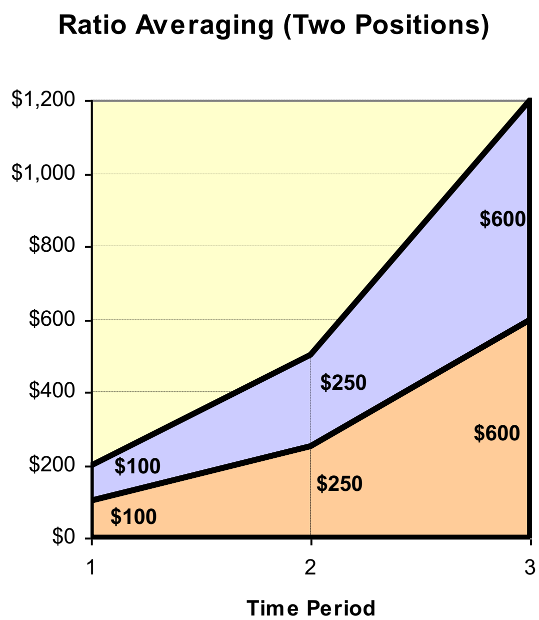
Ratio Averaging (RA) – Again, we use half of our fund ($200) to buy 10 shares of ABC and 10 shares of XYZ to establish initial ratio of 50/50. At time period 2, ABC share rises to $25 and XYZ share declines to $5. With RA, we have the other $200 to invest through rebalancing. Now we have $250 value in ABC, $50 value in XYZ, and $200 new money to invest. In order to maintain 50/50 balance with RA, we want to keep $250 value in each position. We then buy 0 share of ABC ($0) and 40 shares of XYZ ($200). When both stocks reach the target prices of $20, we end up with 10 shares of ABC and 50 shares of XYZ. Thus, the resulting profit is $800 and the value of each position is shown in Figure 4.

In order to compare Ratio Averaging versus Dollar Averaging for buying under practical conditions, let us examine two positions with some typical conditions. When two positions have the same projected rate of return over a given time period, through normalizing prices and without loss of generality, we can assume two positions have the same start prices and the same target price. But for practical reasons, we are considering that the target prices for two positions will not be reached at the same time. Notice that the result can be generalized to multiple positions by using mathematical induction.

The detailed mathematical proof is given in the Appendix. We have proved the following principle on averaging power with multiple positions:

Ratio averaging can yield higher profit than dollar averaging when multiple securities are bought and assumed to have the same projected rate of return over a given time period.

Figure 4



As illustrated in Figure 3 and Figure 4, the results show that RA beats DA in net profit when the target prices are reached for both positions. The reason for the improvement is that through rebalancing, RA helps purchase more shares in lower priced XYZ and less shares in higher priced ABC than DA.

Conclusions

In this paper, efforts are made to form a fundamental framework for comparative study of different averaging strategies. We have studied two important but less understood averaging techniques: Share Averaging (SA) for stock accumulation or liquidation, and Ratio Averaging (RA) for portfolio rebalancing with averaging. A basic mathematical principle about harmonic mean is reviewed and interpreted to assess power of averaging. Since the arithmetic mean and the harmonic mean are equal if and only if prices are equal, which is never the case in any market, we thus conclude the following:

1. *When buying, Dollar Averaging (DA) has more averaging power to make profit than Share Averaging (SA). This is true for up, down, or sideway markets.*

2. *When selling, Share Averaging (SA) has more averaging power to take profit or cut loss than Dollar Averaging (DA). This is true for up, down, or sideway markets.*

We have also studied Ratio Averaging (RA) for trading two positions and proved that if we are given the same projected rate of return for each positon, RA can offer additional averaging power to yield better performance than Dollar Averaging (DA).

For better clarity and without loss of practical correctness, we have developed and demonstrated the results using a few simple sets of market data and assumptions. It is important to emphasize that averaging can also be applied as an effective way to deal psychologically with the volatility of the market. If averaging techniques are further combined with other technical analysis tools, the opportunities for improving trading profit are unlimited. Averaging can help investors or traders to combat the uncertainties inherent in owning any volatile securities.

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intervals over a long period of time can work to your advantage. When values decline, you can buy more shares for the same dollar amount and fewer shares when prices rise. Overall, this may reduce the average cost per share that you pay."
From http://www.mycititrade.com/research/mfunds/mutualfunds_IS_gettingstarted.html

Appendix

The detailed mathematical proof on ratio averaging (RA) versus dollar averaging (DA) for two positions is given as follows.

We denote the total fund for investment as F, and assume that ABC share and XYZ share both have the same start price a and the same target price b.

Let initial investment in each position denoted as C. Consider the case when C = F/4.

Initially:

ABC shares = C/a
XYZ shares = C/a

Since we have projected the target price of ABC and XYZ as given by b, at some time, ABC (or XYZ, similarly) will reach target price b first. We then immediately apply RA or DA with the other position's share price denote as y at that time. We want to assess how the price y will affect the performance of RA versus DA. Without loss of generality, we assume ABC reaches the target price b first.

With Dollar Averaging (DA), we have a fixed dollar amount C to be invested in each position each time:

Total ABC shares after averaging = $C/a + C/b$
Total XYZ shares after averaging = $C/a + C/y$

With Ratio Averaging (RA), we have the ratio of 50/50 fixed for both positions:

Total value before rebalancing = $(C/a)(b+y) + 2C$
Total ABC shares after rebalancing = $[(C/a)(b+y)/2 + C]/b$
Total XYZ shares after rebalancing = $[(C/a)(b+y)/2 + C]/y$

When XYZ also reaches the target price b:

Value of DA = $(C/a + C/b + C/a + C/y)b$
Value of RA = $[(C/a)(b+y)/2 + C] + [(C/a)(b+y)/2 + C]b/y$

We then have

Value of RA = $[(C/a)(b+y)/2 + C] + [(C/a)(b+y)/2 + C]b/y$
$= [(C/a)(b+y)/2+C] + [(C/a)(b+y)/2+C]b/y - (2C/a+C/b+C/y)b + (C/a+C/b+C/a+C/y)b$
$= (C/a)(b+y)/2 + b(C/a)(b+y)/(2y) - 2Cb/a + (C/a+C/b+C/a+C/y)b$
$= (C/a)\ [(b+y)/2 + b(b+y)/(2y) - 2b] + (C/a+C/b+C/a+C/y)b$
$= [C/(2ay)]\ [(by+yy) + b(b+y) - 4by] + (C/a+C/b+C/a+C/y)b$
$= [C/(2ay)]\ [yy + bb - 2by] + (C/a+C/b+C/a+C/y)b$
$= [C/(2ay)]\ (y - b)^2 + (C/a+C/b+C/a+C/y)b$
$> (C/a+C/b+C/a+C/y)b$ (except $y = b$)

Since the target prices will not be reached at the same time, i.e., y is not equal to b, we then conclude that

$[(C/a)(b+y)/2 + C] + [(C/a)(b+y)/2 + C]b/y > (C/a+C/b+C/a+C/y)b$

or

Value of RA > Value of DA